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An Overview of Rotating Stall and Surge Control for Axial Flow Compressors *

Guoxiang Gu[§], Siva Banda[†] and Andy Sparks[†]

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Abstract

Modeling and control for axial flow compression systems have received great attention in recent years. The objectives are to suppress rotating stall and surge, to extend the stable operating range of the compressor system, and/or to enlarge domains of attraction of stable equilibria using feedback control methods. The success of this research field will significantly improve compressor efficiency and thus future aeroengine performance. This paper surveys the research literature and summarizes the major developments in this active research field, focusing on the modeling and control perspectives for rotating stall and surge in axial flow compressors.

1 Introduction

Compressor rotating stall and surge are primary design constraints which effectively reduce engine performance. These are instabilities that arise in the unsteady fluid dynamics. One reason that these unsteady aerodynamic instabilities can lead to large penalties in performance is that they are difficult to predict accurately during design. Feedback control has to be employed to suppress the rotating stall and surge in order to extend the stable operating range and/or to enlarge domains of attraction of stable equilibria for compressor systems and to improve the engine performance.

There were three important developments in this active research field in the past decade. The first was the low-order nonlinear state-space model developed by Moore and Greitzer [29] that captures the nonlinear dynamics of the compressor system through its bifurcation characteristic [4, 28]. The application of classical nonlinear dynamics to rotating stall and surge dynamics motivated the second important development: a simplified approach to rotating stall and surge control based on bifurcation theory. This idea was developed by Abed and his coworkers [2, 4]

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[§] Department of Electrical and Computer Engineering, Louisiana State University, Baton Rouge, LA 70803-5901.

[†] Flight Dynamics Directorate, Wright Laboratory, Wright-Patterson Air Force Base, Ohio, 45433-6553.

and was shown to be effective for implementation in industrial turbomachinery by Nett and his group [13, 14]. Another important development was the linear control method pursued by the MIT group [12, 31, 15]. This survey paper will focus on these three developments on modeling and control of rotating stall and surge in axial flow compressors. Due to space limitations, many details, such as derivation of the nonlinear differential equations and bifurcation diagrams, are omitted, but the related literature is cited. It is hoped that this survey paper will stimulate research interests from the control community in the area of rotating stall and surge control for compressors.

2 Rotating Stall and Surge in Axial Flow Compressors

Axial flow compressors are subject to two distinct aerodynamic instabilities, rotating stall and surge, which can severely limit the compressor performance. Both these instabilities are disruption of the normal operating condition which is designed for steady and axisymmetric flow. The transition from normal compressor operation into rotating stall is depicted in Figure 1 where Φ is the circumferential mean of the flow coefficient ϕ , and Ψ is the nondimensionalized pressure rise. As the flow coefficient through the compressor is decreased (i.e., as the downstream throttle closes in an experiment), the pressure rise increases. This trend continues until the system goes into either rotating stall, surge (deep surge), or both (classic surge).

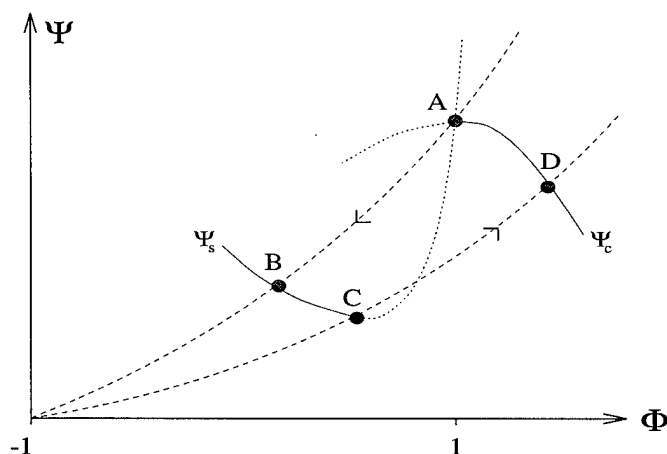


Figure 2: Schematic of compressor characteristic, showing rotating stall

For the case of rotating stall, the lowest flow coefficient at which the compressor can operate with axisymmetric flow is point A, the peak of the characteristic. At lower flows, an abrupt transition occurs into rotating stall (point B). There is a substantial drop in pressure rise and a decrease in flow coefficient (segment A-B). This condition will persist until the flow is increased to point C. Thus there exists a severe 'hysteresis', or range of flow coefficients at which two stable operating conditions exist – steady axisymmetric flow and rotating stall. Once a compressor enters fully developed rotating stall, both rotor and stator blades pass in and out of the stalled

flow causing tremendous stress. Any substantial length of time in this mode can result in excessive internal temperatures due to low efficiency associated with the presence of rotating stall. In addition, an even more serious consequence that can occur in an engine is that the low flow rates obtained during rotating stall can lead to substantial overtemperatures in the burner and turbine [17]. At present, the only remedy to get out of rotating stall is to shut down the engine and restart it again [29].

Rotating stall is a severely non-axisymmetric distribution of axial flow velocity, though steady in an appropriate (moving) reference frame, around the annulus of the compressor, taking the form of a wave or ‘stall cell’, that propagates in the circumferential direction at a fraction of the rotor speed. Surge, on the other hand, is an axisymmetric oscillation of the mass flow along the axial length of the compressor. Deep surge is a mostly axisymmetric oscillation with such a large variation of mass flow that during part of the cycle the compressor operates in reversed flow. The frequency of the surge oscillation is typically an order (or more) of magnitude less than that associated with the passage of rotating stall cells. If surge occurs, the transient consequences such as large inlet overpressures can also be severe. However the circumstances may well be more favorable for returning to unstalled operation by opening either the throttle or internal bleed valves, since the compressor can operate in an unstalled condition over part of each surge cycle. Often surge and rotating stall are coupled (classic surge) although each can occur without the other. For the case of classic surge, the compressor may pass in and out of rotating stall during a surge cycle, with rotating stall characteristics appearing to be quite similar to those obtained during steady-state operation. Thus rotating stall and surge, though coupled, are well defined enough that each can be studied alone for low speed axial flow compressors [31].

Rotating stall and surge are mostly caused by disturbances. Those having largest and most destabilizing effects are: circumferential distortion, planar turbulence, and combustion [22]. All of these types of disturbances present in full-scale aeroengines and are major sources of rotating stall and surge.

- *Circumferential distortion* refers to non-axisymmetric flow patterns that are generated by upstream structures such as bends in inlet duct or boundary layer separation caused by high angle of attack at the engine inlet. The inlet distortion can also be correlated with aircraft angle of attack and yaw angle.
- *Planar turbulence* refers to axisymmetric oscillations in the flow field that are generated, for example, by inlet buzz or ingestion of wakes from nose gear or other aircraft. Planar turbulence is an inherently unsteady flow and has been recognized as an important source of loss in stall margin.
- *Combustion* process introduces large unsteady back-pressure disturbances to the compression system causing steady state operating conditions to exhibit fluctuations in pressure and mass flow large enough to cause the system to diverge.

Thus substantial rotating stall and surge margins are required in the selection of a compressor operating point in order to maintain steady axisymmetric flow. Consequently, compression systems are forced to operate with far less performance operating point than point A, the peak of the compressor characteristic (Figure 1). Even then, with all the above mentioned disturbances present in the worst case, it does not seem possible for the compressor to escape rotating stall and surge unless some control action is taken.

3 Moore-Greitzer Model

Modeling the nonlinear behavior of rotating stall and surge in axial flow compressors has been pursued for about two decades. Greitzer is clearly a pioneer [17, 18]. Although there are many models for rotating stall and surge, the nonlinear model developed by Moore and Greitzer [29] dominates the recent study on rotating stall and surge control because it is a low-order state-space model and it captures the nonlinear features of rotating stall and surge. This section gives a brief review to Moore-Greitzer model with modifications in [2, 28].

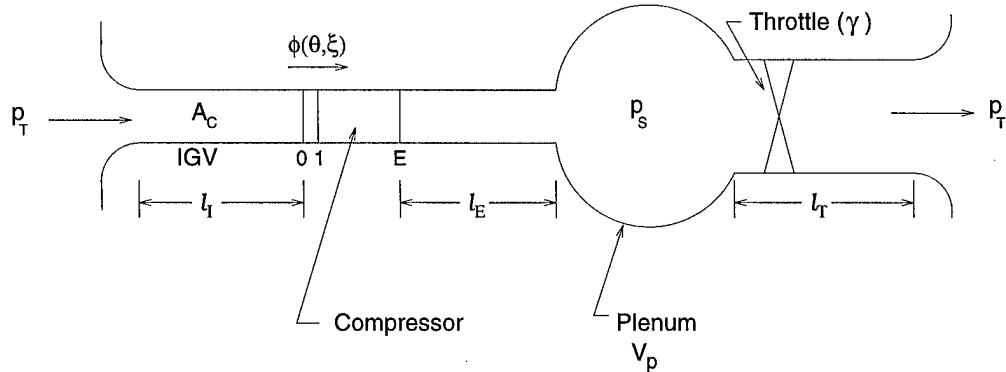


Figure 1: Schematic of compressor showing nondimensionalized lengths

The basic compression system is shown in Figure 2 [29] where P_T is the total pressure upstream of the compressor and P_S the static pressure in the plenum. The flow is assumed to be incompressible in the compressor, though the gas in the plenum is compressible. It is further assumed that the flow is irrotational having no radial variation as it proceeds from an upstream reservoir through the entrance duct to the inlet guide vane entrance at station 0. Such assumptions hold for low speed compressors with high hub-to-tip ratio. The local flow coefficient at station 0 is denoted by $\phi(\xi, \theta)$ with ξ the time nondimensionalized by the rotor speed, and θ the wheel angle. The circumferential mean of the flow coefficient at station 0 is denoted by $\Phi(\xi)$ and is given by

$$\Phi(\xi) = \frac{1}{2\pi} \int_0^{2\pi} \phi(\xi, \theta) d\theta, \quad \phi(\xi, \theta) = \Phi(\xi) + \varphi_0(\xi, \theta),$$

where $\varphi_0 = \varphi(\xi, \theta, 0)$ with $\varphi(\xi, \theta, \eta)$ the flow disturbance. The symbol η denotes the nondimensionalized axial distance that is negative or positive on the left or right of the station 0 respectively. The assumptions on the gas flow imply the existence of a disturbance potential $\tilde{\varphi}(\xi, \theta, \eta)$ that satisfies Laplace's equation

$$\frac{\partial^2 \tilde{\varphi}}{\partial \theta^2} + \frac{\partial^2 \tilde{\varphi}}{\partial \eta^2} = 0, \quad (1)$$

The boundary condition is taken as [28]

$$\frac{\partial \tilde{\varphi}}{\partial \eta}(\xi, \theta, -l_I) = 0. \quad (2)$$

Thus the momentum equation is given by [29]

$$\Psi = \psi_c(\phi) - l_c \frac{d\Phi}{d\xi} - \lambda \frac{\partial \tilde{\varphi}}{\partial \xi} \Big|_{\eta=0} - \frac{1}{2a} \left(2 \frac{\partial^2 \tilde{\varphi}}{\partial \xi \partial \eta} + \frac{\partial^2 \tilde{\varphi}}{\partial \theta \partial \eta} - \mu \frac{\partial^3 \tilde{\varphi}}{\partial \theta^2 \partial \eta} \right) \Big|_{\eta=0}, \quad (3)$$

where $\mu > 0$ is the gas viscosity introduced by [2], l_c the total aerodynamic length of the compressor and duct, λ the exit duct length factor, and $\Psi = (P_S - P_T)/(\rho U^2)$ the pressure rise coefficient with ρ the gas density and U the wheel speed at the mean radius. For the uniform and steady flow, $\psi = \psi_c$, the compressor characteristic curve given by [29]

$$\psi_c(\phi) = \psi_{c0} + H \left[1 + \frac{3}{2} \left(\frac{\phi}{W} - 1 \right) - \frac{1}{2} \left(\frac{\phi}{W} - 1 \right)^3 \right]. \quad (4)$$

For the continuity equation, the isentropic assumption in [29] gives

$$\frac{\dot{m}_c - \dot{m}_T}{V_p} = \frac{d\rho}{dt} = \frac{1}{a_s^2} \frac{d(P_S - P_0)}{dt} \quad (5)$$

where V_p is the volume of the plenum (the gas is compressible inside plenum), a_s is the speed of the sound, and \dot{m}_c the incoming and \dot{m}_T the outgoing mass flow rates respectively. It follows that

$$\Phi - \Phi_T = \frac{V_p U}{a_s^2 A_c} \frac{d\Psi}{d\xi} = 4B^2 l_c \frac{d\Psi}{d\xi}, \quad B := \frac{U}{2a_s} \sqrt{\frac{V_p}{A_c L_c}} \quad (6)$$

with Ψ the circumferential mean of ψ , A_c the compressor duct area, and L_c , the physical dimension of l_c . By assuming parabolic throttle characteristic, (6) is equivalent to

$$l_c \frac{d\Psi}{d\xi} = \frac{1}{4B^2} (\Phi - \Phi_T(\xi)) = \frac{1}{4B^2} (\Phi - F_T^{-1}(\Psi)), \quad \Psi = F_T(\Phi_T) = \frac{1}{2} K_T \Phi_T^2. \quad (7)$$

An important step in the derivation of Moore-Greitzer model is the projection of the PDE (3) to ODE¹. This is accomplished through the use of the spatial harmonic form for disturbance velocity potential [28, 2]:

$$\tilde{\varphi} = \sum_{n=1}^N \left(a_n(\xi) e^{jn\theta} + \bar{a}_n(\xi) e^{-jn\theta} \right) \left(e^{n(\eta+l_I)} + e^{-n(\eta+l_I)} \right), \quad \eta \leq 0, \quad (8)$$

¹PDE stands for partial differential equation and ODE for ordinary differential equation.

because $\tilde{\varphi}$ satisfies Laplace's equation (1) with boundary condition (2). Note that $\{a_n\}$ is defined as the SFC (spatial Fourier coefficient) at station 0, or $\eta = 0$. The number N is dependent on μ . For $\mu = 0$, $N = \infty$, and for $\mu > 0$, N is finite [2]. It follows that

$$\varphi_0 = \sum_{n=1}^N \varphi_{0n}, \quad \varphi_{0n} = W A_n(\xi) \sin(n\theta - r_n \xi), \quad A_n = 4n|a_n| \sinh(nl_I)/W, \quad (9)$$

with r_n a constant by assuming constant speed of the n th harmonic disturbance flow, i.e., constant wave speed. Moore-Greitzer model is now given by

$$\frac{d\Psi}{d\xi} = \frac{W/H}{4B^2} \left[\frac{\Phi}{W} - \frac{1}{W} F_T^{-1}(\Phi) \right] \frac{H}{l_c}, \quad (10)$$

$$\frac{d\Phi}{d\xi} = \left[-\frac{\Psi - \psi_{c0}}{H} + 1 + \frac{3}{2} \left(\frac{\Phi}{W} - 1 \right) \left(1 - \frac{1}{2} J \right) - \frac{1}{2} \left(\frac{\Phi}{W} - 1 \right)^3 \right] \frac{H}{l_c}, \quad (11)$$

$$\frac{dJ_n}{d\xi} = J_n \left(\left[1 - \left(\frac{\Phi}{W} - 1 \right)^2 - \frac{1}{4} J_n \right] \frac{3aH}{W} - \mu n^2 \right) \left(1 + \frac{a\lambda \cosh(nl_I)}{n \sinh(nl_I)} \right)^{-1}, \quad (12)$$

where $J_n = A_n^2$ and $J = \sum_n J_n$. Equation (10) is the same as (7). Equation (11) is obtained by substituting the harmonic form of $\tilde{\varphi}$ into (3) and integrating over a period of 2π . Equation (12) is obtained by the same procedure, but multiplied by $\sin(n\theta - r_n \xi)$ before integration. If instead, $\cos(n\theta - r_n \xi)$ is multiplied before integration, then it can be verified that the wave speed is given by $r_n = n/(2[1 + a\lambda \cosh(nl_I)/\sinh(nl_I)])$.

It is noted that the third order nonlinear state-space model in (10)-(12) includes the models in [29, 2, 28] as special cases. Indeed, by taking $n = N = 1$ and $\mu = 0$, the model used in [28] is obtained. On the other hand, by taking $l_I \rightarrow \infty$, the model used in [2] is obtained. If $n = N = 1$, $\mu = 0$, and $l_I \rightarrow \infty$ are all true, then the original Moore-Greitzer model in [29] is recovered. The third order ODEs in (10)-(12) can also be expanded into $(N + 2)$ th order ODEs by taking $n = 1, 2, \dots, N$ for (12). On the other hand, if \sin and \cos forms are used in (8), then the third order ODE can be expanded into $(2N + 1)$ th order ODE by either taking high order harmonics [2, 3] or by sampling the spatial variable θ uniformly [26]. Thus a much more accurate nonlinear state-space model can be obtained.

For axisymmetric and steady flow, $J = 0$. Thus (11) reduces to $\dot{\Phi} = [-\Psi + \psi_c(\Phi)]/l_c$. Simple calculation shows that linearization about $\Phi = \Phi_e < 2W$ is unstable for some physical parameters due to the positive slope of $\psi_c(\Phi)$, thus rapidly developed into either rotating stall, or surge, or both.

For the case of pure rotating stall, time derivatives in (10)-(12) must vanish (the flow is steady but non-axisymmetric for rotating stall). Equation (12) then requires that J either vanish or have the constant "equilibrium" value

$$J_e = \frac{8W}{3H} \left[\frac{d\psi_c(\Phi)}{d\Phi} - \mu \frac{n^2}{2a} \right] = 4 \left[1 - \left(\frac{\Phi}{W} - 1 \right)^2 - \mu \frac{n^2 W}{3aH} \right].$$

Thus pure rotating stall wave occurs not at the peak (point *A* of Figure 1 at which $\Phi_e = 2W$) for $\mu > 0$. Rather it is slightly on the left of point *A* as μ is small. This is due to the damping effect of the viscosity [2]. Note that pure rotating wave is non-axisymmetric but steady. It travels at constant speed circumferentially. It is also interesting to observe that in order for $J_e \geq 0$, it is necessary for Φ to satisfy either

$$W \leq \Phi \leq \left(1 + \sqrt{1 - \mu \frac{n^2 W}{3aH}}\right) W, \quad \text{or} \quad \left(1 - \sqrt{1 - \mu \frac{n^2 W}{3aH}}\right) W \leq \Phi \leq W.$$

A moment of reflection reveals that $n \leq N \leq \sqrt{3aH/\mu W}$ that is finite for $\mu > 0$ [2]. When the value of J_e is substituted into the right hand side of (11), it gives

$$\dot{\Phi} = [-\Psi + \psi_s(\Phi)]/l_c, \quad \psi_s(\Phi) = \psi_{c0} + H \left[1 - \left(\frac{3}{2} - \mu \frac{W n^2}{aH} \right) \left(\frac{\Phi}{W} - 1 \right) + \frac{5}{2} \left(\frac{\Phi}{W} - 1 \right)^3 \right]. \quad (13)$$

The transition from axisymmetric operating point to pure rotating stall is fast because of the instability that corresponds to the *A-B* path on Figure 1. It is interesting to observe that $\psi_s(\Phi)$ has a negative slope for $0 \leq \Phi \leq (18 - \mu n^2 W)/15$, while $\psi_c(\Phi)$ has a positive slope. Thus pure rotating stall at small Φ is a stable operating point although it is not permissible due to the physical constraint. If the throttle increases, the axisymmetric and steady flow is not recovered along the path *A-B*. Instead, it moves along the path *B-C* (governed by the cubic curve ψ_s , instead of ψ_c) until it reaches the segment of positive slope corresponding to the unstable operating range (specifically $\Phi > (18 - \mu n^2 W)/15$) for certain physical parameters. It then rapidly settles into the axisymmetric and steady flow at point *D*. Hence Moore-Greitzer model captures the nonlinear behavior of rotating stall and explains the so called ‘‘hysteresis’’ in Figure 1.

For the case of pure surge, $\dot{J} = J = 0$. The dynamics is governed by (10) and (11) with $J = 0$. The linearized system may have eigenvalues on either imaginary axis or strictly right half plane. In this case the disturbance wave is axisymmetric but unsteady (deep surge), and can be considered as zeroth mode for the disturbance in (8). A general situation is clearly when rotating stall and surge are coupled. It is emphasized that all these cases are captured by Moore-Greitzer model.

4 Bifurcation Analysis and Nonlinear Control

Moore-Greitzer model predicts the existence of small amplitude disturbance before developing into full rotating stall and surge [29], and it is experimentally validated at MIT [16]. These developments greatly stimulated research interest in employing feedback control methods to suppress the rotating stall and surge when the disturbance has small amplitude. It should be clear that once the compressor runs into fully developed rotating stall, tremendous power is required to get the compressor out of the stall, and active control at this stage is not realistic. Although a linear control method was first suggested in [12], a more profound progress is the use of classic

nonlinear bifurcation analysis and the resulting nonlinear control law for Moore-Greitzer model [2, 3, 4, 7, 13, 14, 25, 27, 28, 35]. This work was initiated by Abed and his group, and aided by McCaughan [27, 28] who gave a more complete bifurcation analysis. The following changes of variables are used:

$$\tilde{J} = \frac{1}{4}J, \quad \tilde{\Phi} = \frac{\Phi}{W} - 1, \quad \tilde{\Psi} = \frac{\Psi}{H}, \quad \tilde{\xi} = \frac{H}{Wl_c}\xi. \quad (14)$$

For simplicity, $N = 1$ is used (Galerkin procedure [29]). Denoting $\beta = 2BH/W$, $\sigma_2 = \mu n^2 W \sigma_1 / (3aH)$, and $\sigma_1 = 3al_c / (1 + a\lambda \cosh(l_I)/n \sinh(l_I))$, (10) – (12) are converted into [28]

$$\dot{\tilde{\Psi}} = \frac{1}{\beta^2} (\tilde{\Phi} - \tilde{\Phi}_T(\tilde{\Psi})), \quad \tilde{\Phi}_T(\tilde{\Psi}) = \gamma \sqrt{\tilde{\Psi}} - 1, \quad \gamma = \sqrt{2H/K_T}/W, \quad (15)$$

$$\dot{\tilde{\Phi}} = -\tilde{\Psi} + \tilde{\Psi}_c(\tilde{\Phi}) - 3\tilde{\Phi}\tilde{J}, \quad (16)$$

$$\dot{\tilde{J}} = \sigma_1 \tilde{J} (1 - \tilde{\Phi}^2 - \tilde{J}) - \sigma_2 \tilde{J}, \quad (17)$$

where the derivatives on the left hand side are with respect to $\tilde{\xi}$, and $\tilde{\Psi}_c = \tilde{\psi}_{c0} + 1 + \frac{3}{2}\tilde{\phi} - \frac{1}{2}\tilde{\phi}^3$, $\tilde{\psi}_{c0} = \psi_{c0}/H$. Linearizing (15) – (17) about an equilibrium point $(\tilde{\Phi}_e, \tilde{\Psi}_e, \tilde{J}_e)$ gives

$$\begin{bmatrix} \dot{\tilde{\Psi}} \\ \dot{\tilde{\Phi}} \\ \dot{\tilde{J}} \end{bmatrix} = L_e \begin{bmatrix} \tilde{\Psi} - \tilde{\Psi}_e \\ \tilde{\Phi} - \tilde{\Phi}_e \\ \tilde{J} - \tilde{J}_e \end{bmatrix}, \quad L_e = \begin{bmatrix} -\beta^{-2}\tilde{\Phi}'_T(\tilde{\Psi}_e) & \beta^{-2} & 0 \\ -1 & \tilde{\Psi}'_c(\tilde{\Phi}_e) - 3\tilde{J}_e & -3\tilde{\Phi}_e \\ 0 & -2\sigma_1\tilde{\Phi}_e\tilde{J}_e & \sigma_1(1 - \tilde{\Phi}_e^2 - 2\tilde{J}_e) - \sigma_2 \end{bmatrix}. \quad (18)$$

For $\mu = 0$, the study in [28] shows that $\tilde{\psi}_{c0} = 4$ is an interesting value. At large values of γ (that is proportional to the cross-sectional area of the throttle), the flow is axisymmetric and steady. Decreasing the value of γ corresponds to reducing the mass flow and causes the flow to loss stability either at the transcritical bifurcation point (for $\tilde{\psi}_{c0} < 4$ or $\tilde{\psi}_{c0} > 4$), or at the pitchfork bifurcation point (for $\tilde{\psi}_{c0} = 4$). The value of γ at which the transcritical bifurcation or pitchfork bifurcation occurs is denoted by γ_c . Clearly the flow corresponding to $J_e = 0$ is stable whenever $\gamma > \gamma_c$ but unstable for $\gamma < \gamma_c$. If $J_e > 0$, a saddle node bifurcation occurs for the case $\tilde{\psi}_{c0} < 4$ at $\gamma_s > \gamma_c$ for which the flow has two branches with one stable if γ increases and the other unstable if γ decreases in the interval of $[\gamma_c, \gamma_s]$ although the flow corresponding to $J_e = 0$ is stable. The bifurcation analysis in [28] gives us a deeper understanding on the hysteresis loop shown in Figure 1. If the flow is initially axisymmetric and steady (i.e., $J_e = 0$), and γ decreases to the value close to γ_c , the matrix L_e in (18) has a pair of complex eigenvalues and one real eigenvalue, all lie strictly on the left half plane. As γ decreases further, the real eigenvalue changes its sign at either the transcritical or pitchfork bifurcation point and $J_e = 0$ loses its stability. Any non-axisymmetric disturbance flow will perturb the compression system to stalled flow: $J_e > 0$. In this case, increasing γ value does not return the compressor to the axisymmetric flow. Rather, the stalled flow will persist until it reaches to the saddle node bifurcation point at $\gamma = \gamma_s > \gamma_c$. The exchange of the stability for the flows corresponding to $J_e = 0$ and $J_e > 0$ at $\gamma \in [\gamma_c, \gamma_s]$ constitutes the hysteresis loop shown in Figure 1. Thus bifurcation analysis gives a

different view on the hysteresis loop and, more importantly, relates it to the parameter γ , that can be used as an actuator through controlling the bleed valves in compression systems [35, 13, 14].

If the γ value decreases further from $\gamma = \gamma_c$, the pair of complex eigenvalues of L_e will also migrate to the right half plane resulting in Hopf bifurcation. As indicated in [28], the Hopf bifurcation is also related to the parameter β , or the B parameter. When $J_e = 0$, the stability of the axisymmetric flow is determined by the reduced second ODE that has a linearized operator

$$L_{er} = \begin{bmatrix} \tilde{\Psi}'_c(\tilde{\Phi}_e) & -1 \\ \beta^{-2} & -\beta^{-2}\Phi'_T(\tilde{\Psi}_e) \end{bmatrix}, \quad \tilde{\Psi}'_c = \frac{d\tilde{\Psi}_c}{d\tilde{\Phi}}, \quad \Phi'_T = \frac{\tilde{\Phi}_T}{d\tilde{\Psi}}. \quad (19)$$

The Hopf bifurcation occurs (that is when L_{er} has a pair of imaginary eigenvalue)

$$\beta = \beta_{HB\tau}(\gamma, \tilde{\psi}_{c0}), \quad \beta_{HB\tau}^2 = \frac{\tilde{\Phi}'_T(\tilde{\Psi}_e)}{\tilde{\Psi}'_c(\tilde{\Phi}_e)} = \frac{1}{3\tilde{\Psi}(1 - \tilde{\Phi}_e)}.$$

Because $\tilde{\Phi}'_T(\tilde{\Psi}) > 0$ for all $\tilde{\Psi} > 0$, the Hopf bifurcation occurs only when $\tilde{\Psi}_c$, or equivalently $\psi_c(\Phi)$, has a positive slope and when $\gamma < \gamma_c$. Moreover the Hopf bifurcations are all supercritical, giving rise to axisymmetric periodic orbits [28]. Using the results on the enforced Van der Pol equation (see also [18]), McCaughan was able to conclude that if β is close $\beta_{HB\tau}$, the limit cycle is unstable and a rotating stall disturbance tends to be amplified, and as β increases from $\beta_{HB\tau}$, the limit cycle grows rapidly and a nonaxisymmetric disturbance tends to be heavily damped, and thus the axisymmetric flow becomes stable, though unsteady. The later situation is associated with deep surge.

When $J_e > 0$ appears at either the saddle node ($\gamma = \gamma_s$, $\tilde{\psi}_{c0} < 4$) or the transcritical bifurcation ($\gamma = \gamma_c$, $\tilde{\psi}_{c0} > 4$), the Hopf bifurcation occurs at $\beta = \beta_{HB}$ when the matrix L_e has a pair of imaginary eigenvalues and a real eigenvalue. For $\beta < \beta_{HB}$, the fixed point $J_e > 0$ is completely stable and that implies rotating stall solution for the compression system. However if $\tilde{\psi}_{c0}$ is approximately smaller than 2.07, the bifurcations are always subcritical regardless of the value of γ ; hence the limit cycle born at the Hopf bifurcation is unstable. When $\tilde{\psi}_{c0}$ is approximately greater than 2.07, the Hopf bifurcation is at first supercritical, and then becomes subcritical as γ is decreased. Since the third eigenvalue is stable, a supercritical Hopf bifurcation implies a completely stable limit cycle. The bifurcation analysis is consistent with the conclusion in [29]: large B parameter (thus large β value) favors surge while small B parameter (thus small β value) favors rotating stall. Thus B parameter is important in determining the nonlinear dynamics of the compression system [17, 18, 29, 13].

Rather than continuing further with the bifurcation analysis as in [28] or as in [2, 4] for the case $\mu > 0$, we turn our attention to the nonlinear control method developed initially in [3, 25, 35] and implemented in [13, 14] that is based on bifurcation characteristic of the compression system. Suppose that the nonlinear model described in (10)-(12) is linearized at $A_e = 0$, $\Psi = \Psi_e$, and

$\Phi = \Phi_e$ (axisymmetric steady flow). Then the linearized system has the form

$$\begin{bmatrix} \dot{\tilde{\Psi}} \\ \dot{\tilde{\Phi}} \\ \dot{A_1} \end{bmatrix} = L_{e0} \begin{bmatrix} \tilde{\Psi} - \tilde{\Psi}_e \\ \tilde{\Phi} - \tilde{\Phi}_e \\ A_1 \end{bmatrix}, \quad L_{e0} = \begin{bmatrix} -\beta^{-2}\tilde{\Phi}'_T(\tilde{\Psi}_e) & \beta^{-2} & 0 \\ -1 & \tilde{\Psi}'_c(\tilde{\Phi}_e) & 0 \\ 0 & 0 & \sigma_1(1 - \tilde{\Phi}_e^2) - \sigma_2 \end{bmatrix}. \quad (20)$$

Thus as discussed earlier, a zero eigenvalue for L_{e0} occurs at $\gamma = \gamma_c$ when

$$\tilde{\Phi}_e^2 = 1 - \frac{\sigma_2}{\sigma_1} \iff \Phi_e = \left(1 \pm \sqrt{1 - \frac{\mu W}{3aH}}\right) W.$$

It follows that the axisymmetric and steady flow loses stability. Any nonaxisymmetric disturbance flow will develop into fully stalled flow. That is, the compression system will exhibit a jump from stable nominal equilibrium when the parameter crosses the critical value γ_c that results in a hysteresis loop of the stable equilibrium. Actually the problem is not the loss of the stability for the axisymmetric and steady flow but the loss of stabilizability with linear control if the throttle is used as actuator [35]. Indeed, write $\gamma = \gamma_c + u$ with u the control actuating signal. Then (20) can be converted into the linearized control system of the form $\dot{x} = Fx + Gu$ with

$$F = \begin{bmatrix} -\beta^{-2}\gamma_c/\sqrt{4\tilde{\Psi}_e} & \beta^{-2} & 0 \\ -1 & \tilde{\Psi}'_c(\tilde{\Phi}_e) & 0 \\ 0 & 0 & \sigma_1(1 - \tilde{\Phi}_e^2) - \sigma_2 \end{bmatrix}, \quad G = \begin{bmatrix} -\beta^{-2}/\sqrt{4\tilde{\Psi}_e} \\ 0 \\ 0 \end{bmatrix}. \quad (21)$$

Clearly the eigenvalue at the origin is not controllable, and thus a linear control method does not work. An important result in [35] is the use of nonlinear control of the form $u = KA_1^2$. It is shown in [35] that with $u = KA_1^2$, the subcritical pitchfork bifurcation at $\gamma = \gamma_c$ can be made into supercritical bifurcation whenever $\Phi_e \neq W$. The elimination of the subcritical bifurcation removes the hysteresis loop and stabilizes the bifurcated equilibrium solutions. It is emphasized that such stabilization is not achievable with linear feedback control. This control law was further improved and successfully implemented in [7, 13, 14] using the measurements on the flow coefficient to estimate the amplitude of the first order spatial harmonic disturbance.

Very recently, a new result was obtained in [24] where backstepping method in [23] was employed for nonlinear control of rotating stall and surge. The nonlinear feedback law in [24] has the form

$$\gamma = \frac{1}{\sqrt{\tilde{\Psi}}} \left\{ 1 + \tilde{\Phi}_e + k_1(\tilde{\Psi} - \tilde{\Psi}_e) + k_2(\tilde{\Phi} - \tilde{\Phi}_e) \right\}. \quad (22)$$

It is interesting to see that with the feedback law in (22), the nonlinear equation (15) reduces to

$$\dot{\tilde{\Psi}} = -\frac{k_1}{\beta^2}(\tilde{\Psi} - \tilde{\Psi}_e) + \frac{1-k_2}{\beta^2}(\tilde{\Phi} - \tilde{\Phi}_e) \quad (23)$$

that is a linear equation. The linearized system has an L_e matrix (as defined in (15)):

$$L_e = \begin{bmatrix} -\frac{k_1}{\beta^2} & \frac{1-k_2}{\beta^2} & 0 \\ -1 & -\frac{3}{2}(1 - \tilde{\Phi}_e^2) & -3\tilde{\Phi}_e \\ 0 & -2\sigma_1\tilde{\Phi}_e(1 - \tilde{\Phi}_e^2) & -\sigma_1(1 - \tilde{\Phi}_e^2) - \sigma_2 \end{bmatrix}.$$

It can be shown that k_1 and k_2 can be chosen such that L_e is a stability matrix, and thus the resulting nonlinear feedback system is locally stable. A more surprising result in [24] is that k_1 and k_2 can be chosen such that the resulting nonlinear feedback system is globally stable. This result is proven using the back stepping method in [23].

5 Linear Perturbation Model and Feedback Control

Another important development in rotating stall control, motivated by Moore-Greitzer model, is the linear control method using inlet guide vanes (IGVs) as actuators. It was suggested first by Epstein, Williams, and Greitzer at MIT [12] to actively damp rotating stall waves at low amplitude by using linear feedback control. Rotating stall can be viewed as the mature form of the rotating disturbance. Damping of the wave would prevent rotating stall from developing, thus stabilizing the flow operating range on the $A-B$ segment (Figure 1). The philosophy is to measure the wave pattern in a compressor and generate a circumferentially propagating disturbance based on those measurements, so as to damp the growth of naturally occurring waves. In the particular implementation described in [31, 32], individual IGVs in upstream are “wiggled” to create the traveling wave velocity disturbance. The flow that the upstream sensors (measured with hot wires) and downstream blade rows see is a combination of naturally occurring instability waves and the imposed control disturbances. As such, the combination of compressor and controller is a different machine from the original compressor — with different dynamic behavior and different operating stability. However in pursuing the linear control method to suppress the rotating stall, it was soon realized that the time lag from the nonaxisymmetric disturbance flow to the disturbance in pressure rise was not taken into consideration in Moore-Greitzer model. Thus a linear perturbation model was developed in [12, 30].

Denote the upstream nonaxisymmetric disturbance flow by $\delta\phi_{\text{upstream}}$ that is measured by hot wire. Denote the nonaxisymmetric disturbance flow at station 0 by $\delta\phi$. Then, by assuming $\delta\phi_{\text{upstream}} = 0$ at $\eta = -\infty$ (this boundary condition is different from that used in (2)), the analysis in the previous section gives the following forms of the nonaxisymmetric disturbance at different stations:

$$\delta\phi_{\text{upstream}} = \delta\phi_{\text{upstream}}(\xi, \theta) = \sum_{n \neq 0} \bar{\phi}_n(\xi) e^{n(\eta - \eta_{hw})} e^{jn\theta} = \sum_{n \neq 0} \phi_n e^{n(\eta - \eta_{hw})}, \quad (24)$$

$$\delta\phi = \delta\phi(\xi, \theta) = \sum_{n \neq 0} \bar{\phi}_n(\xi) e^{-n\eta_{hw}} e^{jn\theta} = \sum_{n \neq 0} \phi_n e^{-n\eta_{hw}}, \quad (25)$$

where η_{hw} denotes the nondimensionalized length from station 0 to the station of the hot wire sensor. Note that $\{\bar{\phi}_n\}$ and $\{\phi_n\}$ are the SFC (spatial Fourier coefficient) at $\eta = \eta_{hw}$ and $\eta = 0$ respectively, and thus they have the same physical meaning as $\{a_n\}$ in (8). The actuator is the densely distributed IGVs for which each individual vane has an incidence of $\gamma(\theta)$, dependent on

θ . The actuating signal is $\delta\gamma$, the variation of the incidence that has a SFC form

$$\delta\gamma = \delta\gamma(\xi, \theta) = \sum_{n \neq 0} \bar{\gamma}_n(\xi) e^{jn\theta} = \sum_{n \neq 0} \gamma_n(\xi). \quad (26)$$

The objective is to synthesize $\delta\gamma$ as a function of $\delta\phi$ such that it damps the disturbance and extends the stable operating range against rotating stall. By ignoring the surge dynamics, and taking IGV into consideration, the PDE (3) is projected into the following single ODE (with $\phi_n = \bar{\phi}_n e^{j\theta}$ and Φ constant):

$$\left[\frac{2}{n} + \nu_t \right] \dot{\phi}_n = \left[\frac{\partial \psi_c}{\partial \phi} + jn\lambda \right] \phi_n - jn\nu_I \Phi \left[\frac{1}{n} + \nu - \frac{\nu_I}{2} \right] \dot{\gamma}_n + \left[\left(\frac{\partial \psi_c}{\partial \gamma} - n^2 \nu_I \lambda \Phi \right) + jn\nu_I \Phi \frac{\partial \psi_c}{\partial \phi} \right] \bar{\gamma}_n,$$

where $\nu_t = \nu + \nu_I$, and parameters of ν, ν_I, λ are the inertia of the IGVs and compressor blades [31, 32]. Note that a different set of parameters from those in (3) is used in [12, 31, 32] and we have kept their notations for ease of the reference. In particular, $\nu = 1/a$ and $\lambda = 1/2a$ are used in Moore-Greitzer model (cf. Section 3). The terms involving $\dot{\gamma}$ and $\bar{\gamma}$ take care of the time lag between the disturbance in flow and disturbance in pressure rise. Parametric representation of the above equation gives the following set of complex-coefficient ODEs:

$$\dot{\bar{\phi}}_n = (\sigma_{RS} + j\omega_{RS}) \bar{\phi}_n + (b_r + jb_i) \bar{\gamma}_n + jg_i \dot{\bar{\gamma}}, \quad (27)$$

where, if we let $\Pi = (\nu_t + 2/n)$,

$$\begin{aligned} \sigma_{RS}(n, \Phi) &= \frac{\partial \psi_c}{\partial \phi_c} / \Pi, \quad \omega_{RS} = n\lambda / \Pi, \\ b_r(n, \Phi) &= e^{n\eta_{hw}} \left(\frac{\partial \psi_c}{\partial \gamma} - n^2 \nu_I \lambda \Phi \right) / \Pi, \\ b_i(n, \Phi) &= e^{n\eta_{hw}} n \nu_I \Phi \frac{\partial \psi_c}{\partial \phi} / \Pi, \\ g_i(n, \Phi) &= -e^{n\eta_{hw}} n \nu_I \Phi \left(\frac{1}{n} + \nu - \frac{\nu_I}{2} \right) / \Pi. \end{aligned}$$

Taking state variables and control inputs as

$$\begin{bmatrix} (x_r)_n \\ (x_i)_n \end{bmatrix} := \begin{bmatrix} \text{Re}(\bar{\phi}_n) \\ \text{Im}(\bar{\phi}_n) \end{bmatrix}, \quad \begin{bmatrix} (u_r)_n \\ (u_i)_n \end{bmatrix} := \begin{bmatrix} \text{Re}(\bar{\gamma}_n) \\ \text{Im}(\bar{\gamma}_n) \end{bmatrix} \quad (28)$$

respectively, it is now straightforward to show that the n -th mode of the SFC for the flow disturbance satisfies the state-space equation

$$\frac{d}{d\xi} \begin{bmatrix} (x_r)_n \\ (x_i)_n \end{bmatrix} = \begin{bmatrix} \sigma_{RS} & -\omega_{RS} \\ \omega_{RS} & \sigma_{RS} \end{bmatrix} \begin{bmatrix} (x_r)_n \\ (x_i)_n \end{bmatrix} + \begin{bmatrix} b_r & -b_i \\ b_i & b_r \end{bmatrix} \begin{bmatrix} (u_r)_n \\ (u_i)_n \end{bmatrix} + \begin{bmatrix} 0 & -g_i \\ g_i & 0 \end{bmatrix} \begin{bmatrix} (\dot{u}_r)_n \\ (\dot{u}_i)_n \end{bmatrix}.$$

Clearly the stability of uncontrolled system is hinged to the sign of derivative of ψ_c with respect to ϕ . A nice feature of the linear perturbation model is the decoupling between different modes of the SFC, and thus a feedback controller can be synthesized for each mode independently.

Different system identification methods were employed in [30, 31, 32] to determine the parameters $(\sigma_{RS}, \omega_{RS}, b_r, b_i, g_i)$ using open-loop frequency response experiments and least-squares fitting. Once the plant model is available, a proportional feedback control law of the form

$$\bar{\gamma}_n = -k_n \bar{\phi}_n, \quad k_n = \text{Re}(k_n) + j\text{Im}(k_n),$$

is employed to enhance the damping ratio. The control signal $\delta\gamma(\xi, \theta)$ is generated according to (26) where the summation is with respect to those modes to be controlled. Clearly this is a state feedback control law by the definition of state variables in (28). Through experimental trials, the best values of the magnitude and phase for k , in terms of maximizing the damping ratio, were obtained, and implemented that achieve the extension of the stable operating range. For using first mode feedback ($n = 1$), the flow coefficient at stall were reduced by 11%. When the second mode ($n = 2$) was also used for feedback, the stall flow coefficient was reduced by 18% [30, 31, 32].

The work reported in [31, 32] also presented a methodology on how to use the general theory and algorithms from system identification and control system design for practical engineering problems. It gave a guideline in the future research work on rotating stall and surge control using linear control theory. However in comparison with the nonlinear control method discussed in the previous section, the linear control method discussed in this section does have drawbacks in using 2D actuator and having large bandwidth for the feedback controller that are contrast to the nonlinear control method, according to [7].

6 Further Developments on Modeling and Control

In the past several years there have been other significant new developments in modeling and control for rotating stall and surge in axial compressors. It is not possible to account every one of them in this survey paper. Thus only those works relating to the modeling and control of compression systems will be described in this section.

As mentioned earlier, Moore-Greitzer model does not take the time lag between flow perturbation across the compressor and the development of perturbation pressure rise into consideration. This problem was investigated further in [19]. An interesting fact is that such time lag stabilizes the high order spatial harmonic modes to a greater extent than the lower ones. As such, only a small number of spatial harmonic modes require control action for the purpose of suppressing rotating stall. Moreover the unsteady development of pressure loss across the compressor was modeled [19, 26]. A Lyapunov stability procedure was also developed in [26] for the analysis of nonlinear phenomena of rotating stall and surge that shed more lights on nonlinear control system design of compression systems.

Another problem with Moore-Greitzer model is that it assumes incompressible flow. Thus assumption holds only for low speed compressor machines, but not for high speed machines. Because engineering interests in rotating stall and surge control are improvement for future aeroengine

performance, it is imperative to study the high speed compressor machines. Control oriented high-frequency turbomachinery modeling, led by Nett, is an important piece of work in this research direction. The models developed in [5, 6] are both inherently high-frequency, exhibiting expected compressor surge and rotating stall phenomena, and well suited for control design due to their relative low complexity and accompanying uncertainty characterization. Notable research work on compressor modeling also include 2D models for both linearized and nonlinear compressible flow. In [20], the model assumes 2D linearized compressible flow in all of the inter-blade passages and ducts, and 1D linearized compressible flow in the blade passages. Using this model, additional modes of the compressor aerodynamic oscillation were identified in [34]. This model was augmented with sensors and actuators in [15] and is now suitable for controller synthesis and analysis. Compressible 2D nonlinear models are currently under development at MIT and UTRC, including 3D nonlinear model. The removal of the incompressible flow assumption is important because axial compressors used in aeroengines typically have high rotor speeds and large pressure ratios.

In the past, rotating stall control and surge control were studied separately. For instance, the two relatively successful control system design methods (nonlinear and linear ones) discussed in the previous two sections address only rotating stall. Although surge control was not reviewed here, interested readers will find [8] helpful. In general, rotating stall and surge are not separable physically. Both are flow disturbances with surge as the zeroth order mode and rotating stall the non-zeroth order modes of the spatial harmonic disturbance. It is natural to expect the feedback controller to have the dual role of suppressing both rotating stall and surge. This research problem has received increasing attention [13] recently.

For rotating stall and surge control, many different sensor-actuator schemes are available [33, 21]. Recently there is an increased interest in using air injectors as actuators and pressure transducers as sensors because of the initial work in [10]. In [11] a set of three air injectors are equally placed on the sensor ring in front of the rotor. The basic strategy of the control algorithm was to sense the location and magnitude of a stall cell with three equally spaced dynamic pressure transducers and apply pulses of air to locations of decreased pressure. In [9, 15], the use of jet actuators is more sophisticated where a circumferential array of 12 jet actuators is paced 63 mm upstream of the compressor face with down stream static pressure as measured output. The linearized compressible flow model developed in [20] was modified in [15] to develop a rational transfer matrix relating the actuator input and sensor output determined by system identification methods, and the LQG control methodology is then used to synthesize the feedback controller for active suppression of rotating stall. Regarding the use of sensors, flow rate measurements with hot wires are less reliable than static pressure measurements with pressure transducer. Moreover hot wires are very delicate and difficult to survive the hostile environment such as high speed axial compressors.

Up to present, three different types of actuators have been implemented and tested exper-

imentally: inlet guide vanes [31, 32], air injection [10, 15, 11], and bleed valves [7]. Because inlet guide vanes involve 2D actuation and requires large torque motors, this actuation scheme is not likely to be used in aeroengines due to the limit of weight and power supply. On the other hand bleed valves involve only 1D actuation and thus admit considerable advantages over inlet guide vanes. Moreover the research work in using bleed valves as actuators has been advanced significantly for the last several years that results in bifurcation based nonlinear feedback control. Hence the research work along this line is quite mature. A less developed control scheme is the use of air injector as actuators where only logic type or linear control has been studied so far. For the use of sensors, static pressure measurements have a more promising future. More and more research work tend to employ pressure transducers [15, 11, 1] because they are less expensive and more reliable. Thus nonlinear feedback control with jet actuators and pressure sensors will be an important research direction in the near future.

7 Conclusion

This survey paper reviews the recent developments for rotating stall and surge control in compression systems. In spite of the significant developments in rotating stall control, there are still many problems to be studied. The most important one is the control system design for suppressing both rotating stall and surge for high speed compressors. We hope this survey paper will spur the research interests from the control community in this active research field. Because of the vast literature in this field, we apologize in advance if some of the important work was overlooked.

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